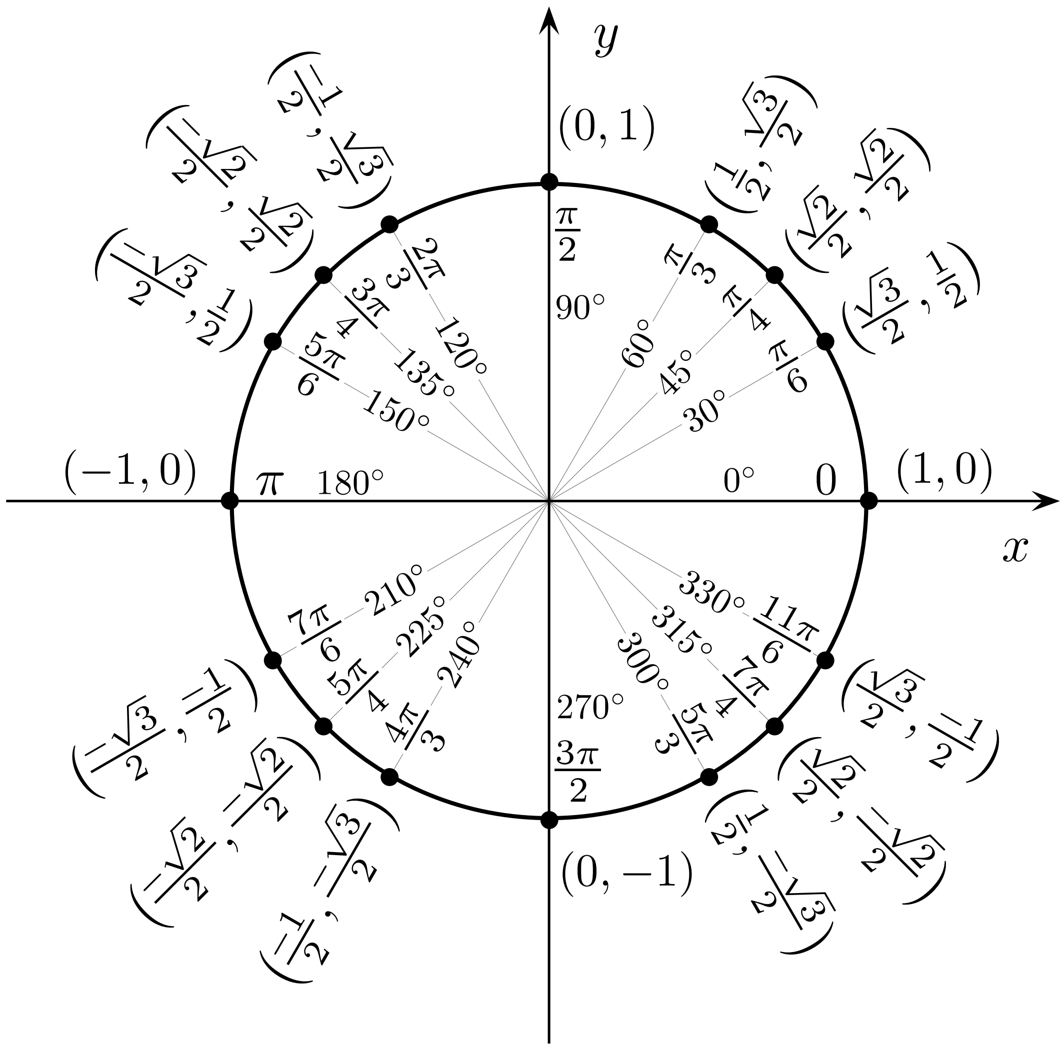
Trigonometry

The Unit Circle

The unit circle is a circle of radius 1 centered at the origin, the point $(0,0)$. Given a point (x,y) on the perimeter of the unit circle, there is a ray[1] from the origin to (x,y). This ray together with the ray from the origin to $(1,0)$ forms an angle θ (measured counterclockwise) such that $x=\cos\theta$ and $y=\sin\theta$. The converse is also true.

For example to find the cosine of $\frac{\pi}{6}$, find the ray that forms $\frac{\pi}{6}$ with the ray from the origin to $(1,0)$. The coordinate of the intercection of the ray and the perimeter of the circle is $(\frac{\sqrt{3}}{2},\frac{1}{2})$ and therefore $\cos\frac{\pi}{6}=\frac{\sqrt{3}}{2}$.



[1] Given two distinct points A and B the ray from A to B is the set of points C in the line that contains A and B such that A is not strictly between B and C.

Converting Between Degrees and Radians

To convert between degrees and radians we use the fact that π radians$= 180°$. To convert an angle α from radians to degrees, multiply α by $\frac{180}{\pi}$. So α radians $= \alpha \cdot \frac{180}{\pi}$ degrees. Similarly to convert an angle β from degrees to radians, multiply β by $\frac{\pi}{180}$. So β degrees $= \beta \cdot \frac{\pi}{180}$ radians.

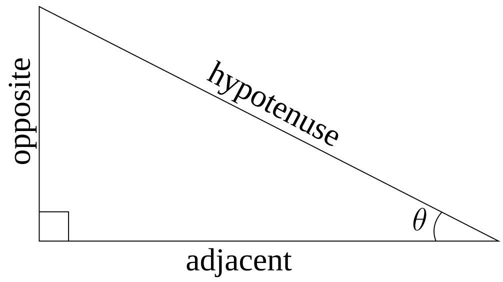
Sine, Cosine and Tangent

In the case of a right triangle, given a non-right angle θ

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

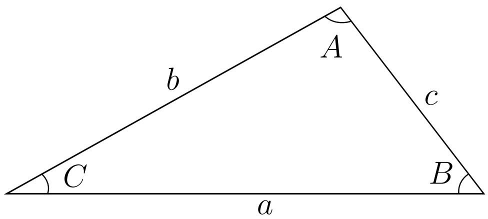


Laws of Cosines and Sines

Given any triangle with sides a, b, c and opposite angles A, B, C respectively, we have

Law of Cosines: $c^2 = a^2 + b^2 - 2ab\cos C$

Law of Sines: $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$



Trigonometric Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = 1 - 2\sin^2\theta$$