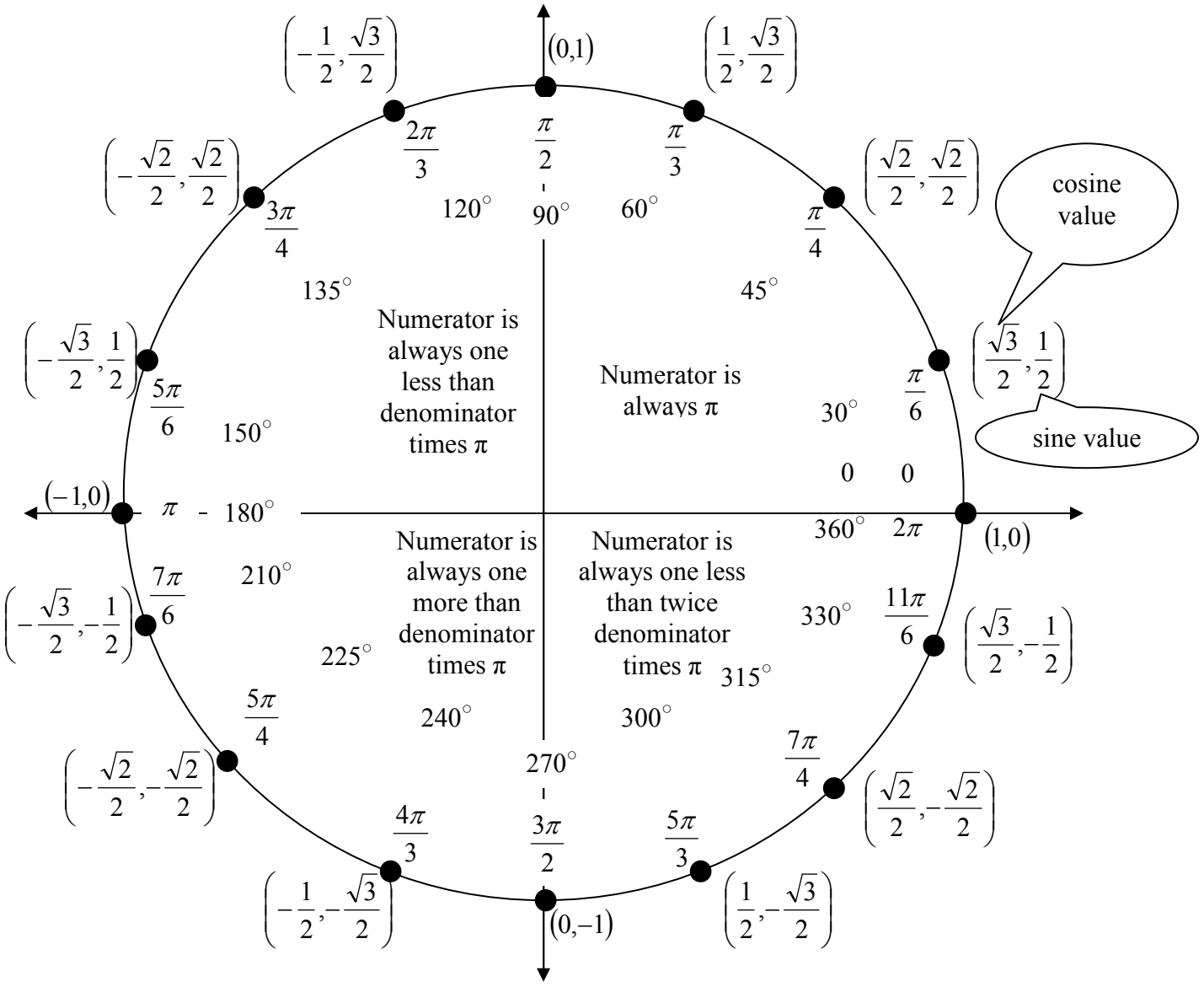
Unit Circle and Reference Angles



Some hints when dealing with radians.

Value of Denominator	Reference Angle
$\frac{?}{6}$	30°
$\frac{?}{4}$	45°
$\frac{?}{3}$	60°

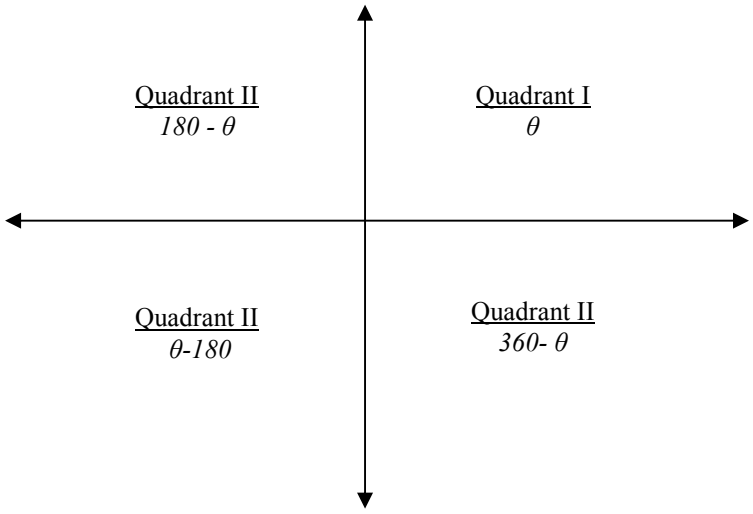
A. Reference Angles:

Example: Draw $60°\left(\frac{\pi}{3}\right)$ in Quadrant II.

1. (a) We are in the Quadrant II with a reference angle of 60 degrees. The angle in radians has a denominator of 3. Thus, the angle in Quadrant II is 120 degrees or 2π/3.

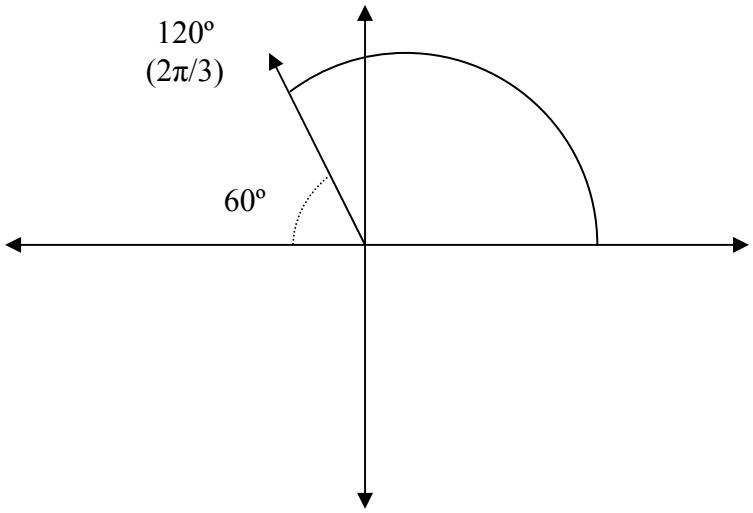
(b)Formulas for Reference Angles:

Note: Let θ be the reference angle given.



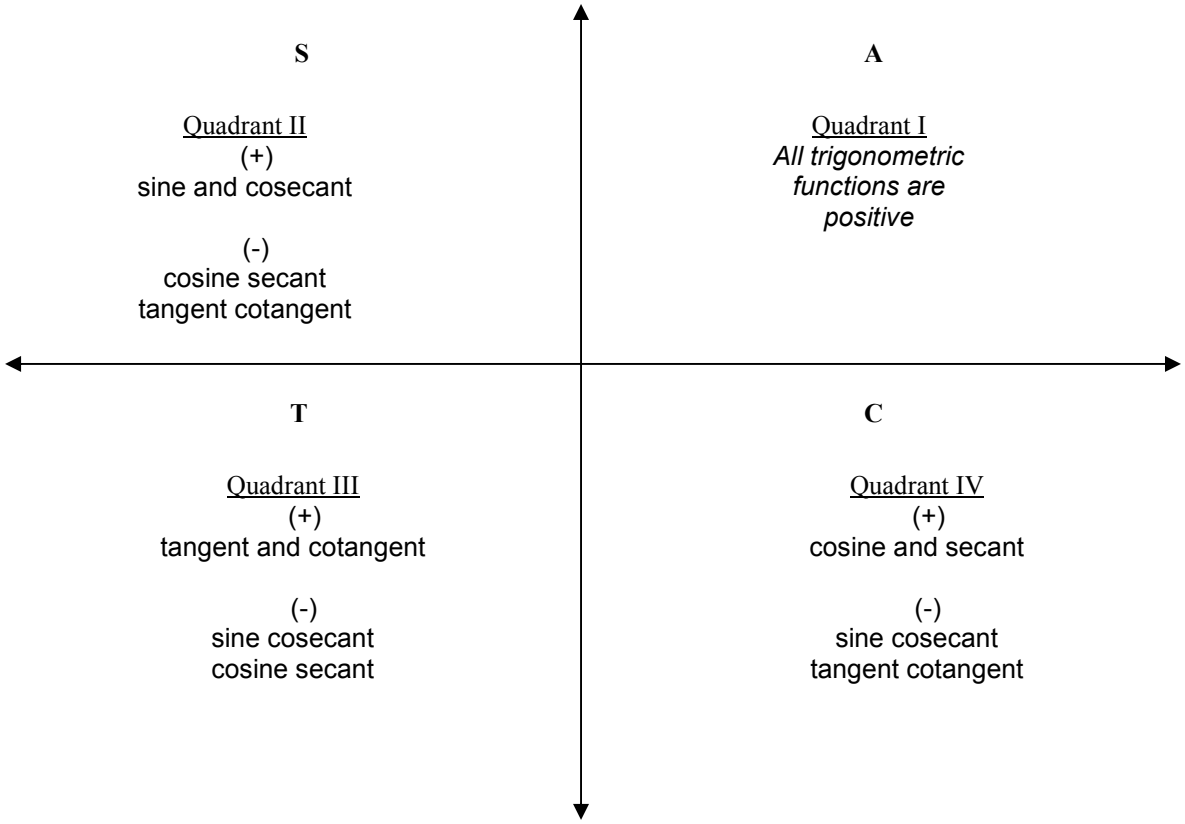
In this example, we in are in Quadrant II. Thus, the angle is 180 degrees-60 degrees = 120 degrees.

Solution:



Note: ***Knowing reference angles speeds up things considerably.***

Positive and Negative Quadrants



*A*ll trigonometric functions are positive in Quadrant I

S*ine and cosecant are positive in Quadrant II*

T*angent and cotangent are positive in Quadrant III*

C*osine and secant are positive in Quadrant IV*

Note:* ***This information is used in conjunction with reference angles.*

B. Using Reference Angles To Solve Trigonometric Equations.

Example: $Solve: \sin 2\theta = -\frac{\sqrt{2}}{2}$.

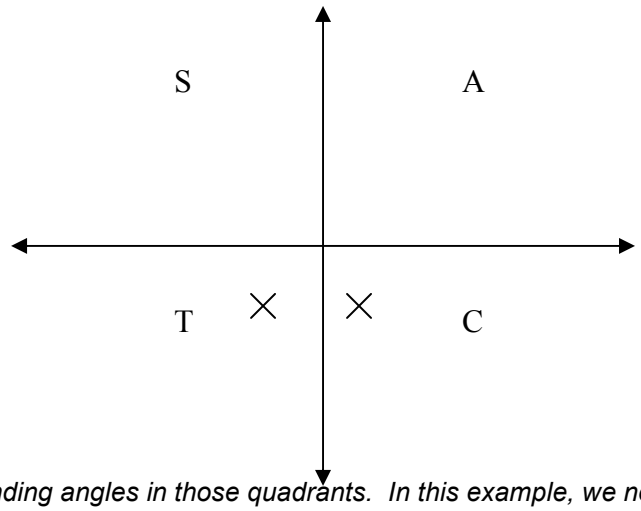
*To solve this equation, take the **inverse sine** (**arcsin**) of both sides.*

$$\sin^{-1}(\sin 2\theta) = \sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$$

*Note: **2θ means you have to make two revolutions around the unit circle. nθ determines the number of revolutions.***

Steps:

1. Determine which quadrants your desired angles lie in. In this example, sine is negative in Quadrants III and IV.



2. Find corresponding angles in those quadrants. In this example, we need an angle in Quadrant III and an angle in Quadrant IV which has a sine of $-\frac{\sqrt{2}}{2}$. *The reference angle is 45 degrees which means the angles have a denominator of four.*

First time around:

--Quadrant III (angle in radians is one more than denominator)

$$\frac{5\pi}{4}$$

--Quadrant IV (angle in radians is 1 less and twice denominator)

$$\frac{7\pi}{4}$$

Second time around (just add 2π to the previous angles):

--Quadrant III (angle in radians is one more than denominator)

$$\frac{5\pi}{4} + 2\pi = \frac{13\pi}{4}$$

--Quadrant IV (angle in radians is 1 less and twice denominator)

$$\frac{7\pi}{4} + 2\pi = \frac{15\pi}{4}$$

Solution: $\frac{5\pi}{4}, \frac{7\pi}{4}, \frac{13\pi}{4}, \frac{15\pi}{4}$