Introduction

In this section, the lessons focus on the concepts of area, perimeter, and volume. The eighth grade lessons focus on understanding and applying the Pythagorean Theorem.

These lessons form an outline for your ARI classes, but you are expected to add other lessons as needed to address the concepts and provide practice of the skills introduced in the *ARI Curriculum Companion.*

Some of the lessons cross grade levels, as indicated by the SOL numbers shown below. Such lessons help students connect the content from grade to grade and to accelerate.

Standards of Learning

5.8 The student will
- a) find perimeter, area, and volume in standard units of measure;
- b) differentiate among perimeter, area, and volume and identify whether the application of the concept of perimeter, area, or volume is appropriate for a given situation;

6.10 The student will
- c) solve practical problems involving area and perimeter;

8.10 The student will
- a) verify the Pythagorean Theorem; and
- b) apply the Pythagorean Theorem.

8.11 The student will solve practical area and perimeter problems involving composite plane figures.

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✱ SOL 5.8a

Lesson Summary

Students determine the perimeter and/or area of given figures. (60 minutes)

Materials

Boxes of toothpicks
"Perimeter and Area" worksheets
"Reflection" worksheets

Vocabulary

perimeter. The distance around a object. It is a measure of length.

area. The number of square units needed to cover a surface or figure.

Warm-up

Have the students write the definitions of *perimeter* and *area* in their own words, and then ask them to share their definitions with the class. Refine the students' definitions to form official class definitions of these terms.

Lesson

1. Give half a box of toothpicks to each pair of students.
2. Ask each pair to determine the perimeter of a desktop, using the toothpicks as the unit of measure. Have each pair write their finding on the board. (Note: All desktops measured must be the same size.)
3. Have the students compare the various measurements written on the board. If they vary, is the difference between the largest and the smallest a big one? Or are all perimeter measures close to the same? Ask the students to explain *how* they made their measurements. List on the board the techniques the students used, such as
   - laying toothpicks end-to-end around the edge of the desk and then counting the toothpicks
   - counting the number of toothpicks around one length and one width and then doubling this number
   - placing one toothpick over and over end-to-end around the edge.
4. Distribute copies of the "Perimeter and Area" worksheet, and point out how the techniques previously discussed can be used to solve problems 1 through 4. Allow students time to complete the perimeter calculations, and assist students who need help.
5. When students have finished these first four problems, ask them to work in pairs to find the area of a desktop in toothpicks. Allow plenty of time for exploration.
6. Once pairs have completed the task, ask them to write a description of *how* they found the area—what procedure they followed.
7. Have a class discussion when the pairs have completed the task, asking pairs to describe how they found the area. Did they find a length and width in toothpicks and then multiply the two measurements? Did they cover the desktop in "toothpick squares" and count? List all student responses.
8. Have students solve problems 5 through 8 on the worksheet, using a method discussed in step 7. Give assistance as necessary.
9. Introduce finding the area of a triangle, as follows:
   - Draw a rectangle on the board, and measure the length and width in toothpicks.
   - Mutliply the length times the width to calculate the area in toothpick squares.
   - Draw a diagonal to cut the rectangle in half, creating two triangles.
   - Ask the students for the area of one of these triangles. They should recognize that it is half the area of the rectangle.

10. From this demonstration, ask the students how to find the area of *any* triangle, using a rectangle. (The base and height of any triangle can be equal to the length and width of a rectangle, as in the example on the board. If you multiply the base by the height of any triangle and then divide by 2, you get the area of one of the two triangles that form a rectangle with the same length and width. This can be expressed algebraically as $A = \frac{b \times h}{2}$ or $A = \frac{1}{2}bh$).
11. Assign number 9 and 10 on the worksheet, and give assistance as needed.

Reflection

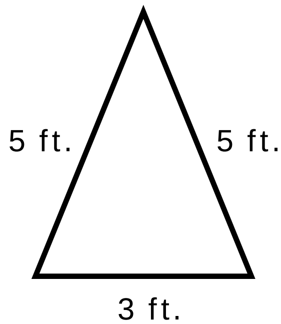
Have students complete the "Reflection" worksheet.

Name: ______________________

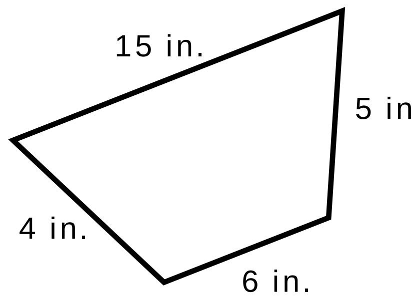
Perimeter and Area

Find the *perimeter* of the following polygons.

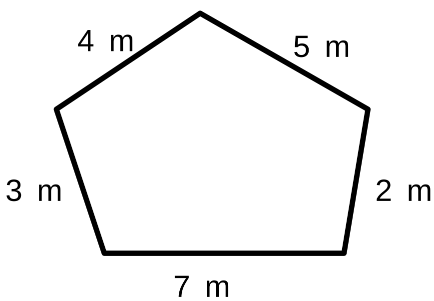
1.



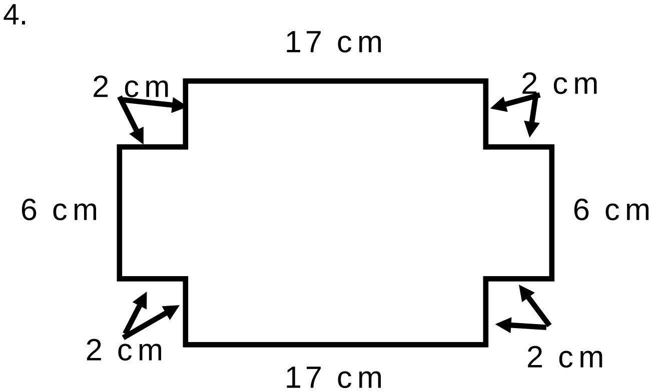
2.



3.

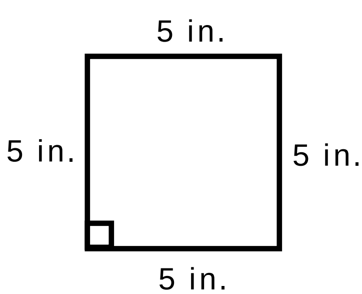


4.

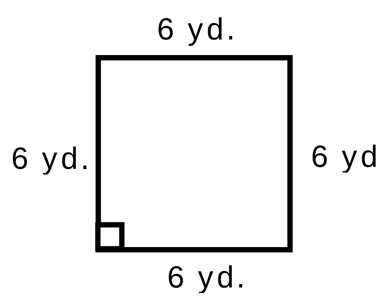


Find the *area* of the following figures.

5.



6.



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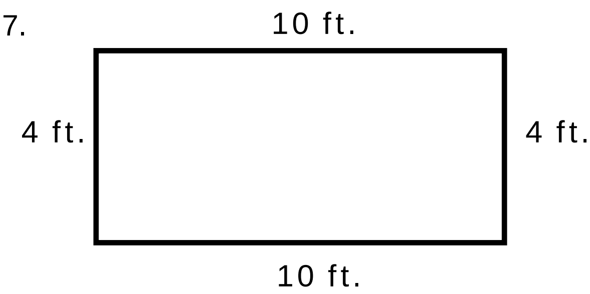
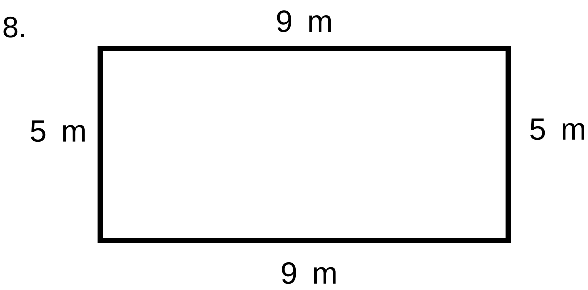
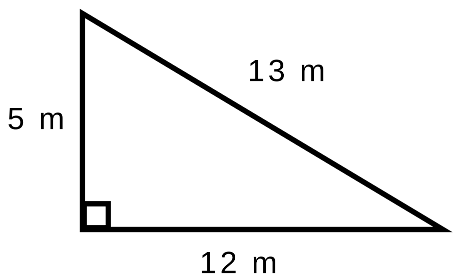


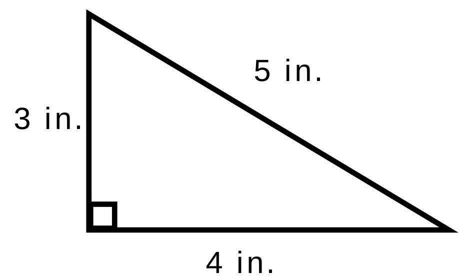
8.



9.



10.

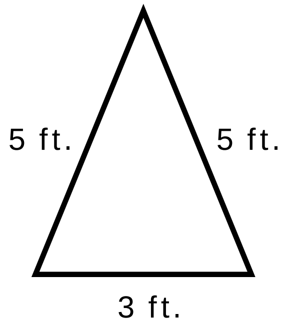


Name: <u>ANSWER KEY</u>

Perimeter and Area

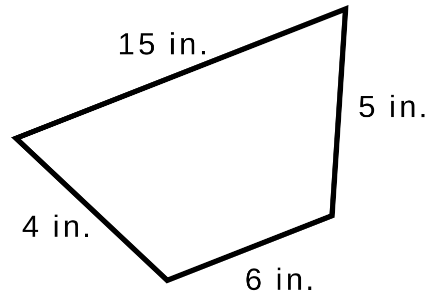
Find the *perimeter* of the following polygons.

1.



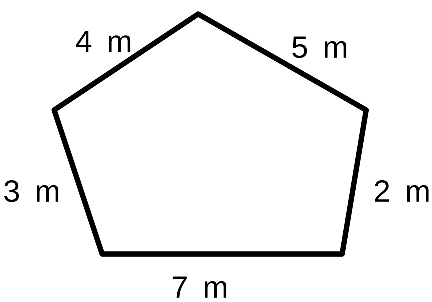
13 ft.

2.



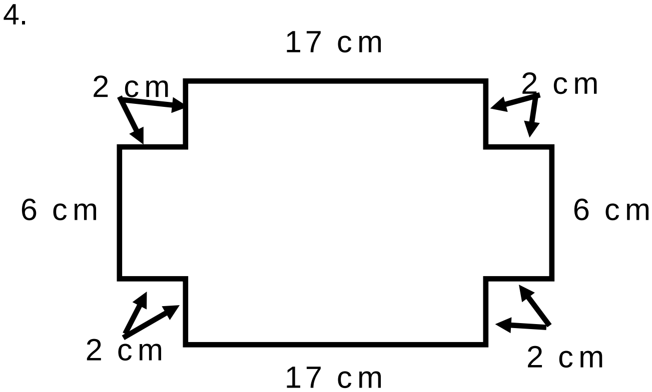
30 in.

3.



21 m

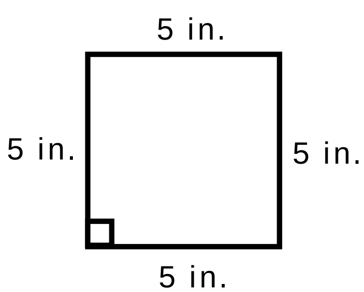
4.



62 cm

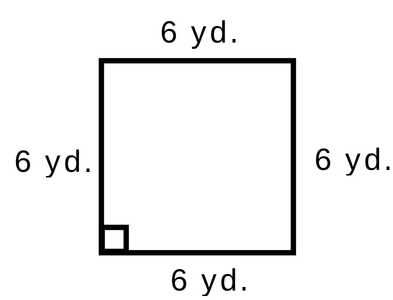
Find the *area* of the following figures.

5.



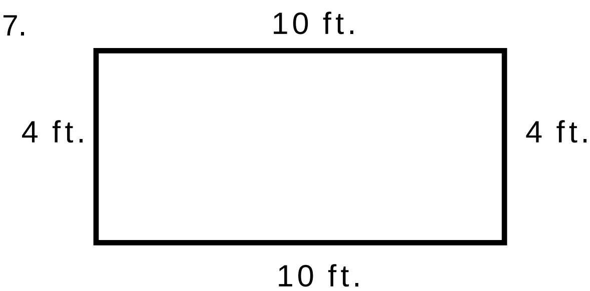
25 in.^{2}

6.



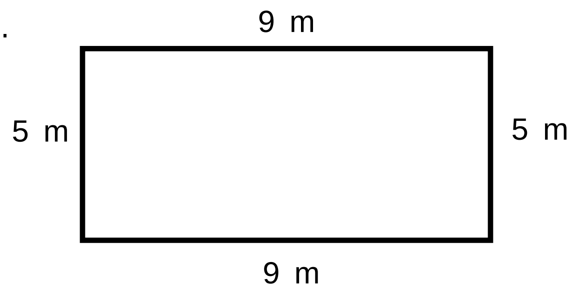
36 yd.^{2}

7.



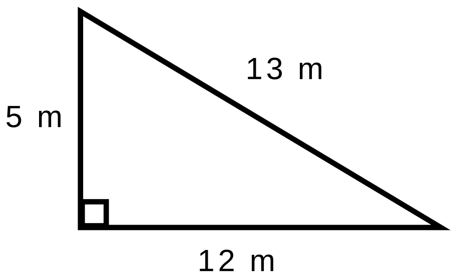
$40\ \text{ft.}^2$

8.



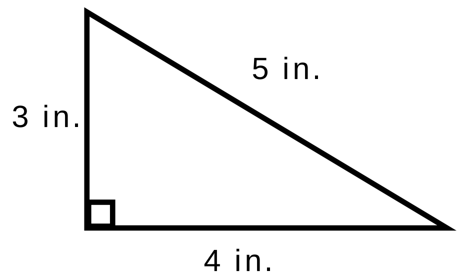
$45\ \text{m}^2$

9.



$30\ \text{m}^2$

10.



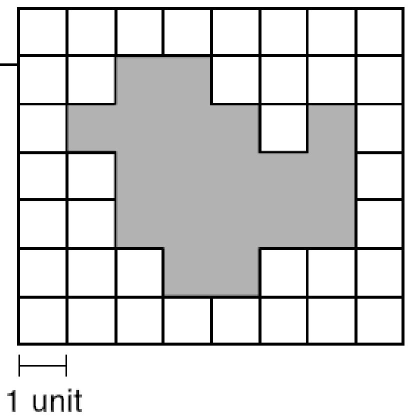
$6\ \text{in}^2$

Name: ________________________

Reflection

The question shown at right is a released SOL test question.

What is the perimeter of the shaded figure on the grid below?



F 20 units
G 21 units
H 22 units
J 24 units

1. What are you asked to find?

   ____

2. How can you find the perimeter?

   ____

   ____

3. Which answer do you choose?

   ______

4. Explain your reasons for selecting the answer you chose.

   ____

   ____

   ____

Name: ANSWER KEY

Reflection

The question shown at right is a released SOL test question.

1. What are you asked to find?

   The perimeter of the figure

2. How can you find the perimeter?

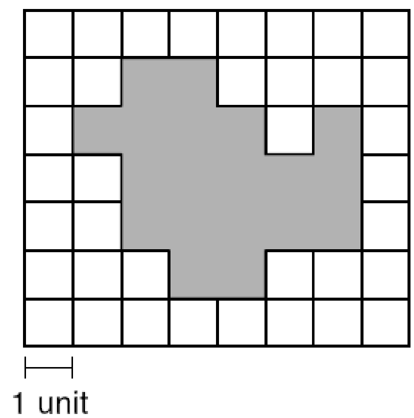
   Add the lengths of the sides, or count the number of blocks to go around the figure

3. Which answer do you choose?

   J

4. Explain your reasons for selecting the answer you chose.

What is the perimeter of the shaded figure on the grid below?



F 20 units
G 21 units
H 22 units
J 24 units

✱ SOL 5.8a

Lesson Summary

Students estimate and find the area of polygons. (25 minutes)

Materials

3 inch x 3 inch construction paper squares
“Estimating Area” worksheets
“Reflection” worksheets

Vocabulary

area. The number of square units needed to cover a surface or figure.

Warm-up

Make 3 inch x 3 inch squares from construction paper, and give one to each student. Ask students to *estimate* the smallest number of these squares that would be needed to exactly cover the top of a desk or table. Encourage them to use the square to approximate the size of the desktop. Ask them what they could do if the squares do not exactly cover the top of the desk. (Use *pieces* of squares, such as right triangles, to approximate the area.)

Lesson

1. Distribute copies of the “Estimating Area” worksheet. As students look at problem 1, explain that a square has been placed on the grid, and ask for suggestions about ways to estimate the area of the square. (Students should respond with suggestions to count the number of 1-unit squares inside the large square and to add the odd-shaped pieces together to make more 1-unit squares.)
2. Have the students try the remaining shapes on their own. Provide individual assistance as needed.
3. Compare answers as a class. Did everyone arrive at the same answers? Why, or why not?
4. Conduct a class discussion about why the ability to estimate area is an important skill to have.

Reflection

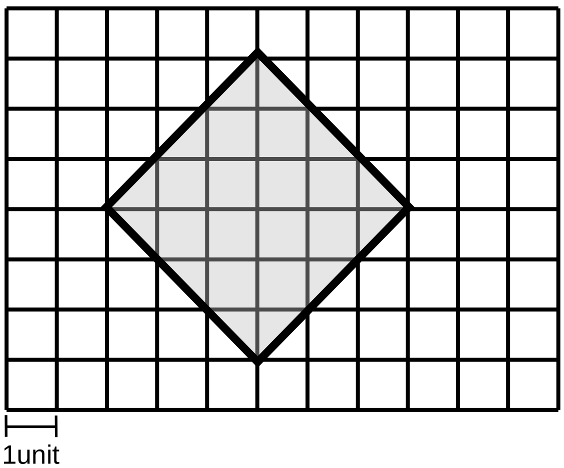
Have students complete the “Reflection” worksheet.

Name: ________________________

Estimating Area

Use the grid to estimate the area of the figure to the nearest whole number.

1.



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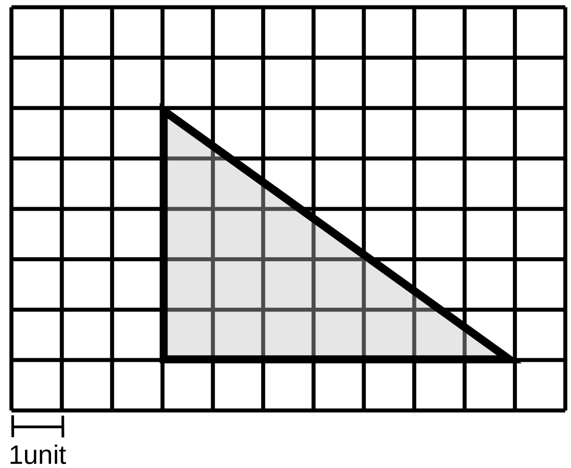
2.



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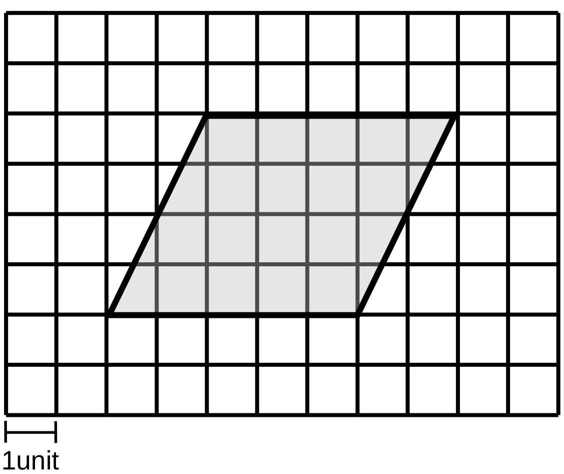
3.



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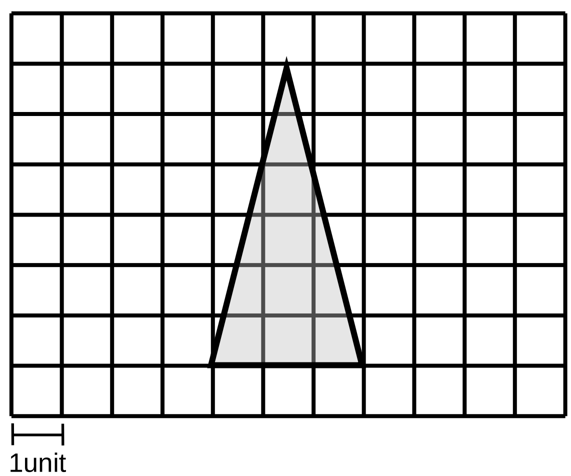
4.



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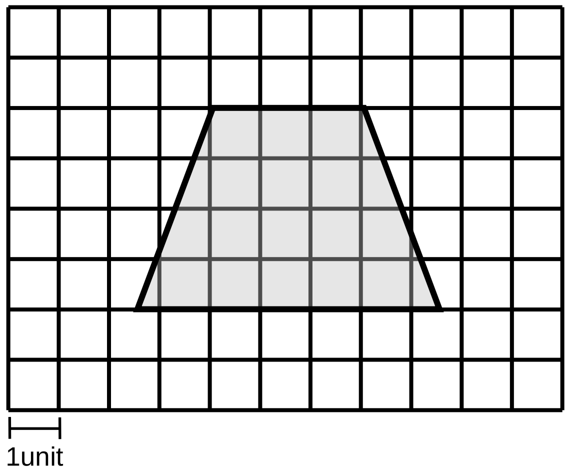
5.



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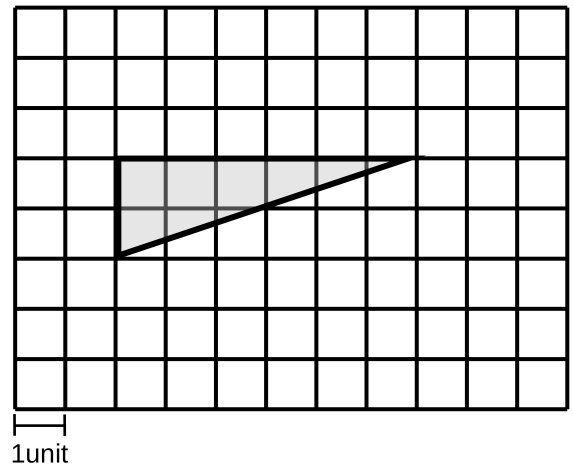
6.



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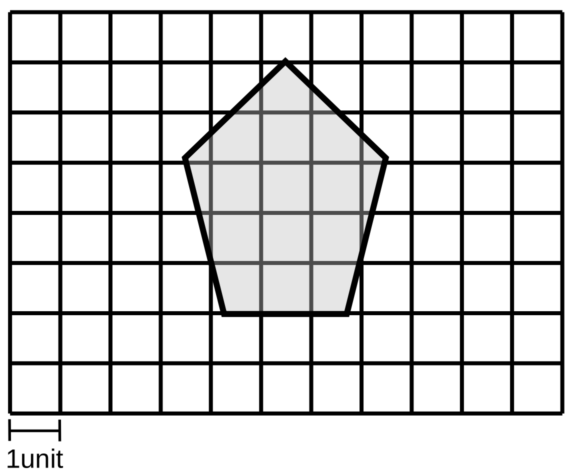
7.



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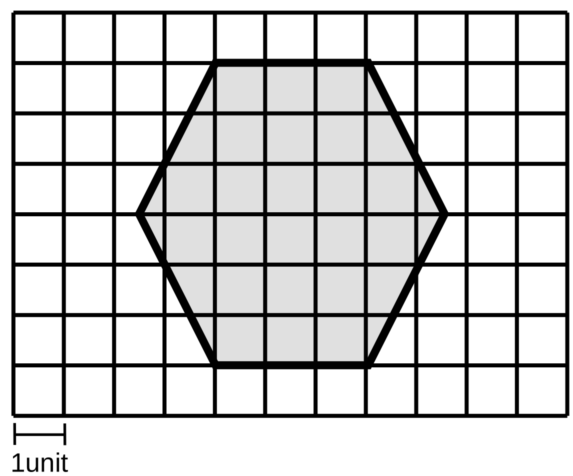
8.



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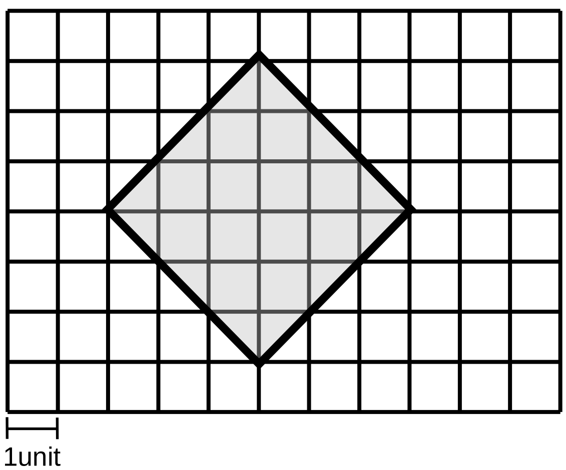
Name: ANSWER KEY

Estimating Area

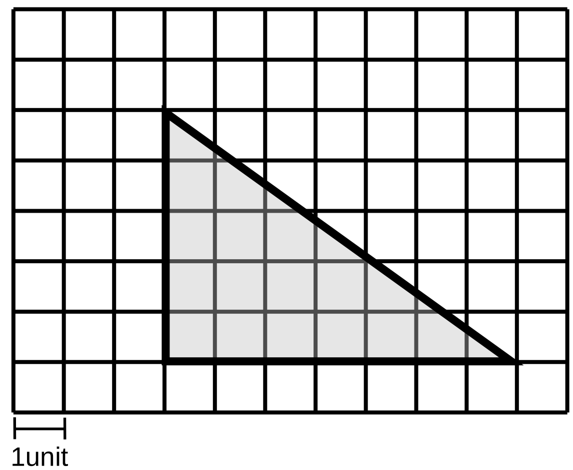
Use the grid to estimate the area of the figure to the nearest whole number.

1.



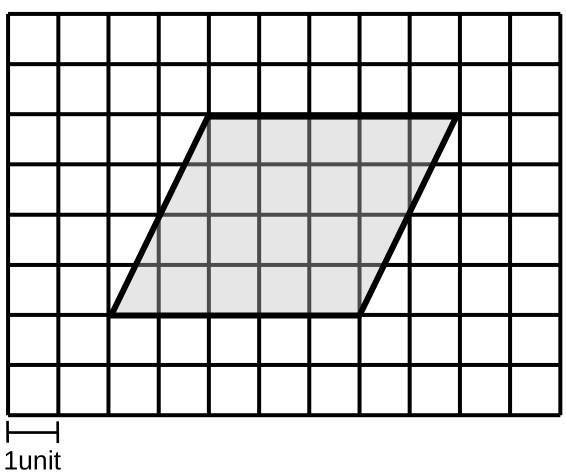
18 square units

2.



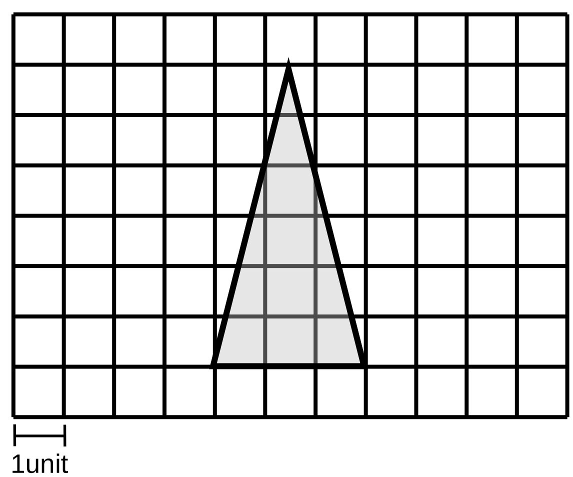
17 square units

3.



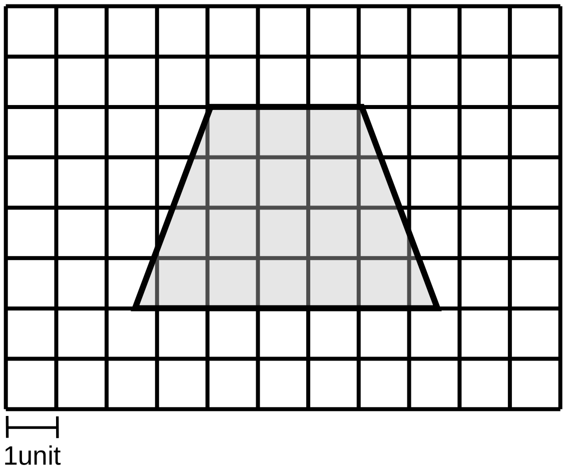
20 square units

4.



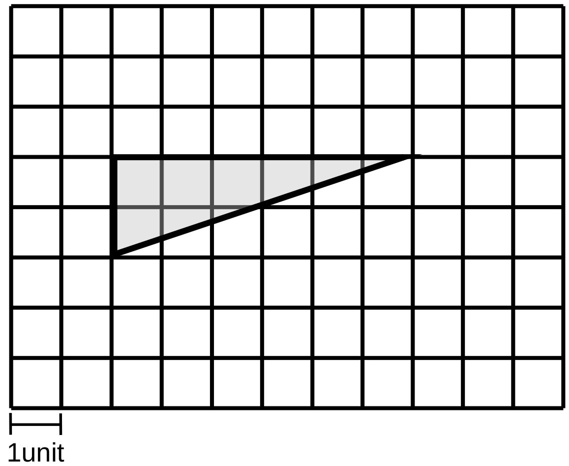
9 square units

5.



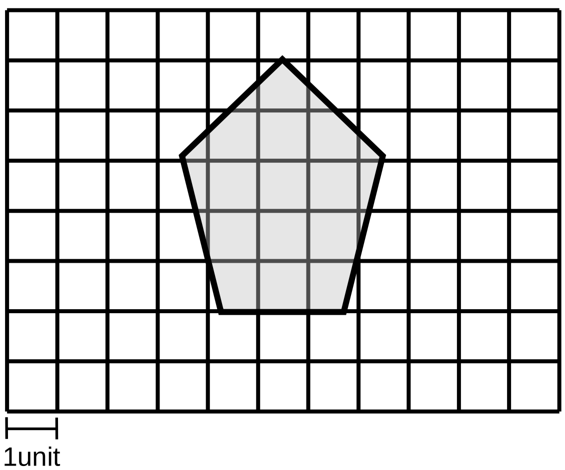
18 square units

6.



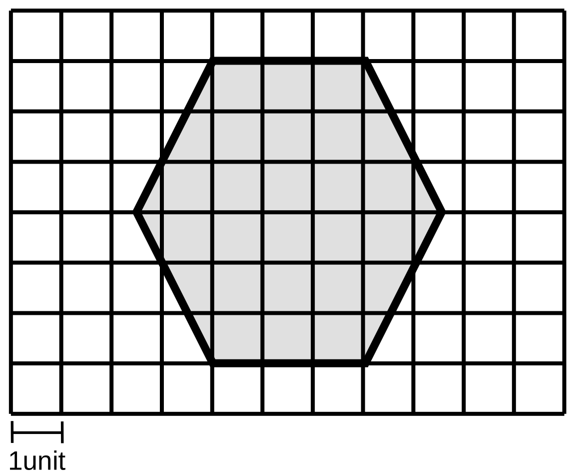
6 square units

7.



13 square units

8.



27 square units

Name: ______________________

Reflection

The question shown at right is a released SOL test question.

1. What are you asked to find?

____

2. How can you find the area?

____

____

3. Which answer do you choose?

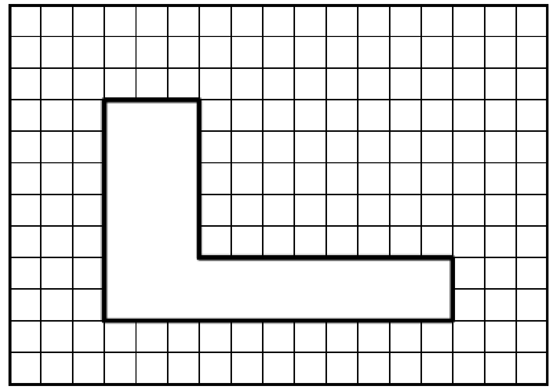
____

4. Explain your reasons for selecting the answer you chose.

____

____

____



In this scale drawing of a molding, each square represents 1 square inch. What is the area of the molding?

F 24 sq in.
G 34 sq in.
H 35 sq in.
J 37 sq in.

Name: ANSWER KEY

Reflection

The question shown at right is a released SOL test question.

1. What are you asked to find?

   The area of the figure

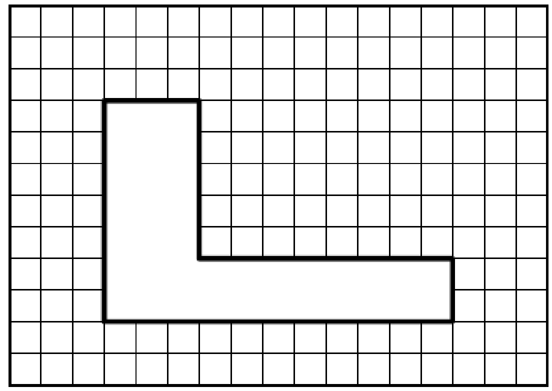
2. How can you find the area?

   Count the number of squares the figure covers

3. Which answer do you choose?

   J

4. Explain your reasons for selecting the answer you chose.



In this scale drawing of a molding, each square represents 1 square inch. What is the area of the molding?

F 24 sq in.
G 34 sq in.
H 35 sq in.
J 37 sq in.

✱ SOL 5.8b

Lesson Summary

Students determine whether a given situation is an application of the concept of perimeter, area, or volume. (20 minutes)

Materials

"Warm-up" worksheets
"You Make the Decision" worksheets

Vocabulary

perimeter. The distance around an object. It is a measure of length.

area. The number of square units needed to cover a surface or figure.

Warm-up

Distribute copies of the "Warm-up" worksheet, and allow students time to solve the problems. Review the answers when students have completed the work.

Lesson

1. Ask students for examples of situations in which someone may need to measure around the exterior of an object or location. (Installing a fence, placing a border around a room) List their responses on the board. Explain that these are examples of situations in which it is important to be able to calculate perimeter.
2. Ask students for examples of situations in which someone may need to cover a surface. (Painting a wall, mowing grass) List their responses on the board. Explain that these are examples of situations in which it is important to be able to calculate area.
3. Ask students for examples of situations in which someone may need to know how much a container holds. (The number of cubes that will fit in a box, the amount of water that will fit in a container) List their responses on the board. Explain that these are examples of situations in which it is important to be able to calculate volume.
4. Distribute copies of the "You Make the Decision" worksheet, and allow students time to complete it. Provide assistance where needed.
5. After checking everyone's work, have the students play "Hot Seat." Allow one student to sit in a chair (the hot seat) in front of the room. In turn, have students read the examples of area, perimeter, and volume they wrote. If the student in the hot seat can answer 3 (or 5 or 6 or however many you designate) in a row, he/she is retired and does not have to answer any more questions. If a student "stumps" the student in the hot seat, he/she has the honor of sitting in the hot seat. The game continues until there are no more new questions to ask.

Reflection

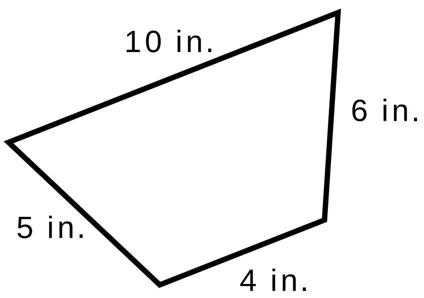
Have the students write about a real-life time in their life when they or someone they knew used a perimeter, area, or volume calculation to complete a task.

Name: ________________________

Warm-up

Find the *perimeter* of the following polygons.

1.



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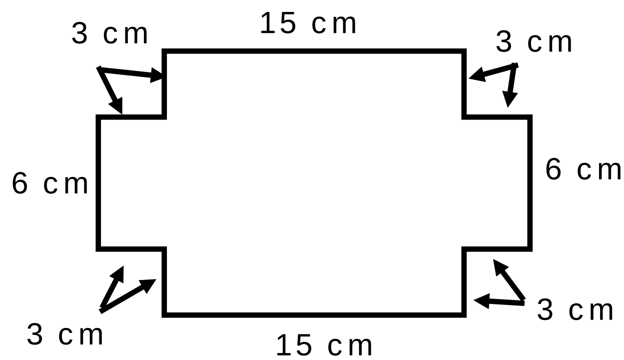
2.



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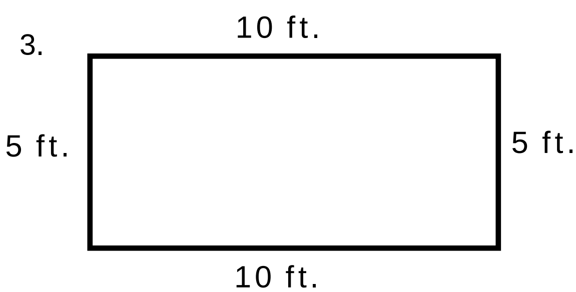
Find the *area* of the following figures.

3.



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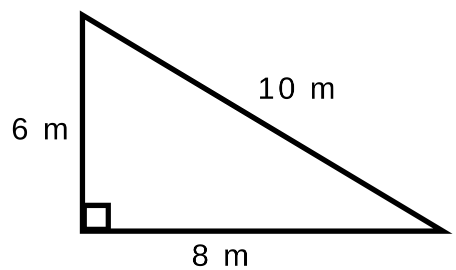
4.



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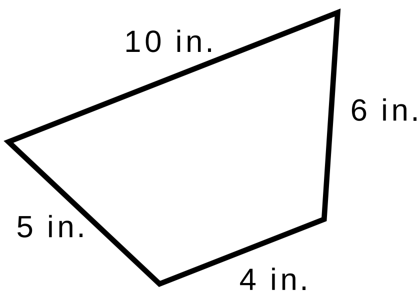
Name: ANSWER KEY

Warm-up

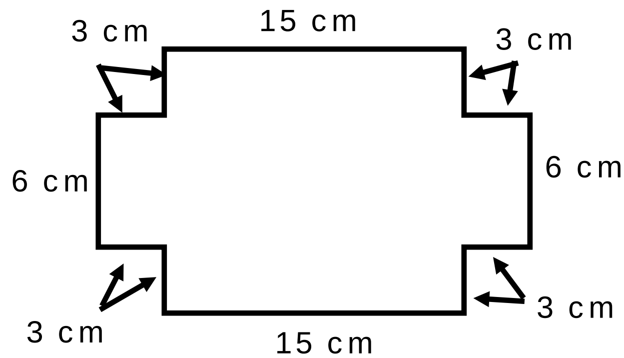
Find the *perimeter* of the following polygons.

1.



25 in.

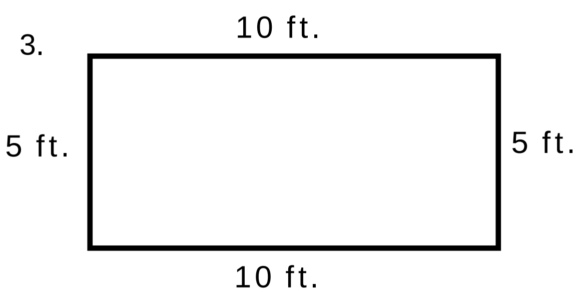
2.



66 cm

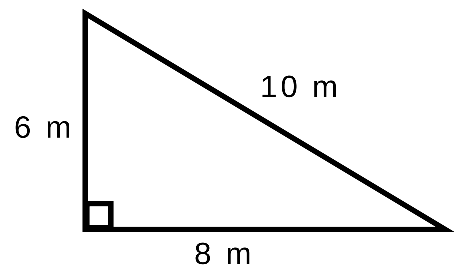
Find the *area* of the following figures.

3.



50 ft^2

4.



24 cm^2

Name: ______________________

You Make the Decision

Decide whether each of the following situations is an example of area, perimeter, or volume.

1. The distance around a city block in New York City ____________
2. The amount of wallpaper needed to cover a wall of your bedroom ____________
3. The length of string needed to wrap around a large box ____________
4. The amount of new carpeting to carpet the family room ____________
5. The amount of gasoline needed to fill a car's gas tank ____________
6. The size of a cover for a swimming pool ____________
7. The amount of water needed to fill the swimming pool ____________
8. The length of fence needed to surround a swimming pool ____________
9. The amount of grass in an athletic field ____________
10. The length of wood needed to add a baseboard to the kitchen ____________

Write three examples of your own:

Area: ____

____

Perimeter: ____

____

Volume: ____

____

Name: ANSWER KEY

You Make the Decision

Decide whether each of the following situations is an example of area, perimeter, or volume.

1.	The distance around a city block in New York City	perimeter
2.	The amount of wallpaper needed to cover a wall of your bedroom	area
3.	The length of string needed to wrap around a large box	perimeter
4.	The amount of new carpeting to carpet the family room	area
5.	The amount of gasoline needed to fill a car's gas tank	volume
6.	The size of a cover for a swimming pool	area
7.	The amount of water needed to fill the swimming pool	volume
8.	The length of fence needed to surround a swimming pool	perimeter
9.	The amount of grass in an athletic field	area
10.	The length of wood needed to add a baseboard to the kitchen	perimeter

Write three examples of your own:

Area: ____

____

Perimeter: ____

____

Volume: ____

____

✱ SOL 8.10a

Lesson Summary

Through an investigation, students discover the relationships in the Pythagorean Theorem. (60 minutes)

Materials

Markers	Rulers
Scissors	Yellow and blue construction paper cut into 6 x 6 inch squares
Glue

Vocabulary

right triangle. A right triangle has one right angle.

hypotenuse. The side of a right triangle located opposite the right angle; always the longest side in a right triangle.

legs. The two sides of a right triangle that form the right angle.

Pythagorean Theorem. $a^2 + b^2 = c^2$, where c is the hypotenuse and a and b are the legs of a right triangle. The square of the length of the hypotenuse equals the sum of the squares of the legs (altitude and base).

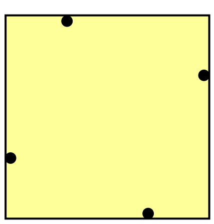
congruent figures. Two figures that are exactly the same size and shape.

Warm-up

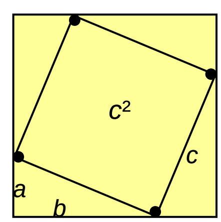
Draw three different sized squares on the board, and label the length of one side of each square with a variable. Ask the students to copy the squares and write the area of each square inside the square. (For example, if a square has side length c, the area of the square is c^2.) Discuss the students' responses.

Lesson

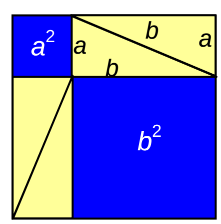
1. Give each student a marker, glue, scissors, a ruler, a yellow 6 x 6 inch square, and a blue 6 x 6 inch square. Have students check that the yellow and blue squares are the same size and shape—are congruent—by placing one on top of the other.



2. Have students use a ruler and the marker to locate and put a dot one-third of the way along each side of the yellow square, starting on one side and continuing clockwise (shown at left). Demonstrate the steps on the board as you give the directions.
3. Have students connect the dots, forming a square and four triangles (shown at right). Ask students how the four triangles compare with each other. (They are congruent.)



4. Have students label one side of the large square c and write the area of this square (c^2) in the center.
5. Have students label the sides of the four triangles a and b, being careful to be consistent: side a should be the shortest side of each triangle.
6. Have students cut out the center square, thereby making five shapes: one square with area c^2 and four congruent triangles with sides a and b. Have them test for congruence by placing the triangles on top of each other.
7. Have students rearrange their four triangles as shown at right and glue them to the blue square in this pattern.



8. Ask students to name the areas of the two smaller blue squares formed by this arrangement of the triangles. (The area of the smaller square is a^2; the area of the other square is b^2.)
9. Ask the students what must be true about the areas a^2 and b^2 together. (The areas a^2 and b^2 added together are equal to the area c^2 that was identified earlier. Because the

original yellow and blue squares were the same size, when the four yellow triangles were moved to the blue square, the areas left uncovered on the blue square must be equal to what was left of the original yellow square, that is, area c^2.)

Reflection

Have the students affix their final construction paper pieces to a sheet of notebook paper. Have them explain at the bottom of the page how these pieces demonstrate the equation $a^2 + b^2 = c^2$.

✱ SOL 8.10a

Lesson Summary
Students use the Pythagorean Theorem to verify right triangles. (15 minutes)

Materials
"Warm-up" worksheets
"Verifying the Pythagorean Theorem" worksheets
Calculators

Vocabulary
right triangle. A triangle containing a 90° angle.

hypotenuse. The side of a right triangle located opposite the 90° angle; always the longest side in a right triangle.

legs. The two sides of a right triangle that form the 90° angle.

Pythagorean Theorem. $a^2 + b^2 = c^2$, where c is the hypotenuse and a and b are the legs of a right triangle.

Warm-up
Have students complete the "Warm-up" worksheet. Provide assistance as needed.

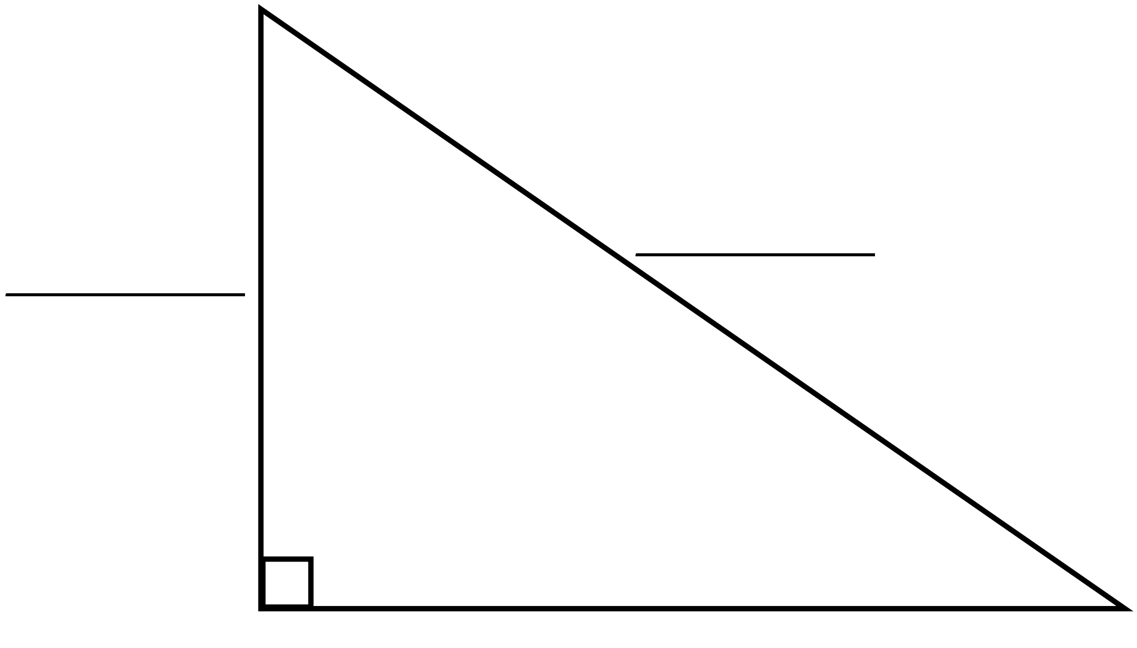
Lesson
1. Distribute copies of the "Verifying the Pythagorean Theorem" worksheet.
2. Have the students label each triangle's hypotenuse and legs, based on appearance.
3. Remind the students that they do not know for sure whether these triangles are right triangles. To find out, they can apply the Pythagorean Theorem. Have them substitute the lengths of the sides into the equation. Use example 1 as a class demonstration, if needed.
4. Once the students are comfortable with the procedure, have them complete the worksheet, and assist students who need help.

Reflection
Have students write in their own words how to verify whether a triangle is a right triangle or not, using the Pythagorean Theorem.

Name: ______________________

Warm-up

Label the following right triangle, using the terms *hypotenuse* and *leg*

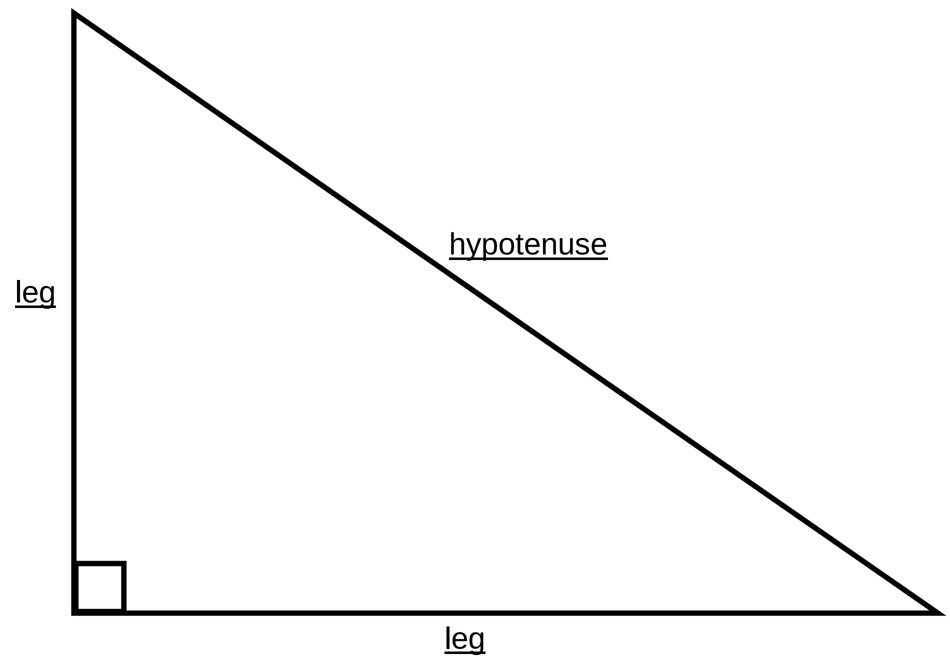


What is the Pythagorean Theorem? ____________________________

Name: ANSWER KEY

Warm-up

Label the following right triangle, using the terms *hypotenuse* and *leg*



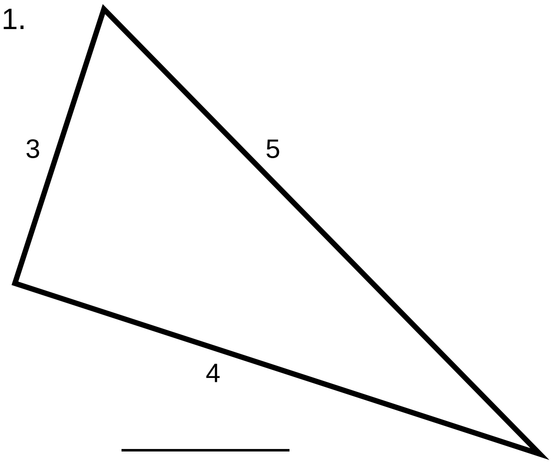
What is the Pythagorean Theorem? $a^2 + b^2 = c^2$

Name: ________________________

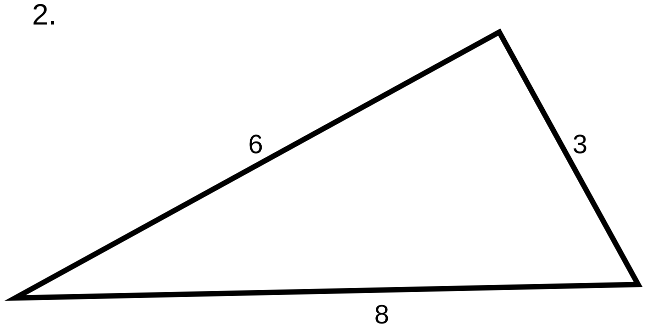
Verifying the Pythagorean Theorem

Use the Pythagorean Theorem to determine whether each triangle is a right triangle. On the line below each triangle, write *Yes* if it is a right triangle or *No* if it is not. For right triangles, label the legs and the hypotenuse. Show your work on a separate sheet of paper.

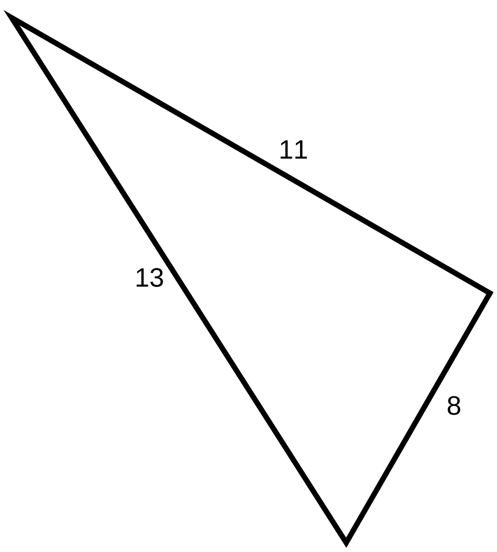
1.



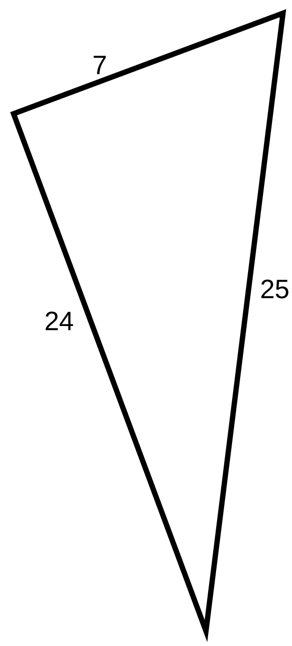
2.



3.



4.

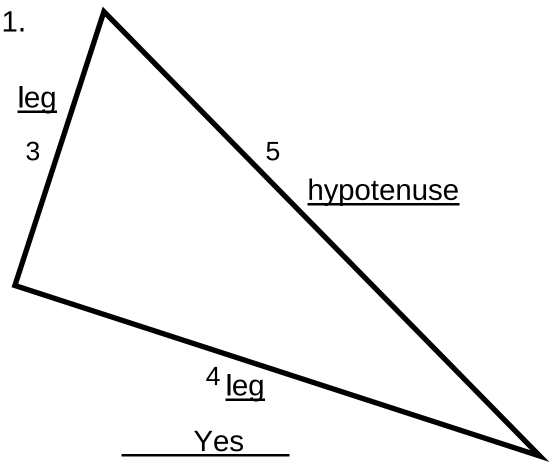


Name: <u>ANSWER KEY</u>

Verifying the Pythagorean Theorem

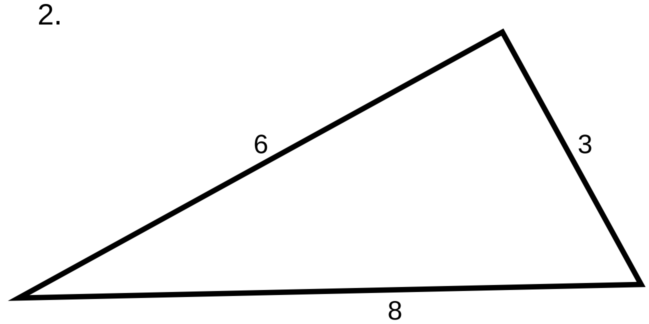
Use the Pythagorean Theorem to determine whether each triangle is a right triangle. On the line below each triangle, write *Yes* if it is a right triangle or *No* if it is not. For right triangles, label the legs and the hypotenuse. Show your work on a separate sheet of paper.

1.



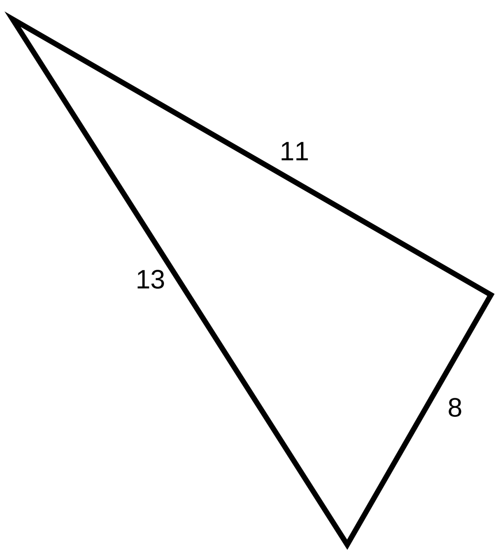
_____Yes___

2.



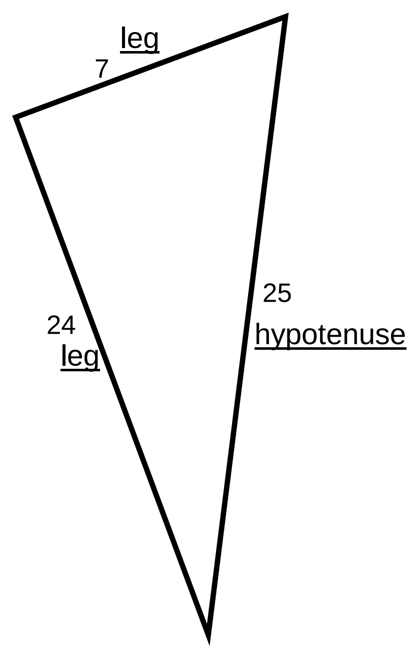
_____No____

3.



_____No____

4.



_____Yes___

✱ SOL 8.10b

Lesson Summary

Students use the Pythagorean Theorem to find the missing measure of any side of a right triangle, given the measures of the other two sides. (30 minutes)

Materials

"Warm-up" worksheets
"Pythagorean Theorem" worksheets
Calculators
"Reflection" Worksheet

Vocabulary

right triangle. A triangle containing a 90° angle.

hypotenuse. The side of a right triangle located opposite the 90° angle; always the longest side in a right triangle.

legs. The two sides of a right triangle that form the 90° angle.

Pythagorean Theorem. $a^2 + b^2 = c^2$, where *c* is the hypotenuse and *a* and *b* are the legs of a right triangle.

Warm-up

Distribute copies of the "Warm-up" worksheet, and assist students individually as needed in completing it.

Lesson

1. Distribute copies of the "Pythagorean Theorem" worksheet.
2. Have the students label each triangle's hypotenuse as *c* and the legs as *a* and *b*.
3. Ask students how they can find the missing length of any side of a triangle, using the given lengths of the other two sides. They should suggest writing the Pythagorean Theorem equation $a^2 + b^2 = c^2$ and then substituting the given lengths for *a* and *b*.
4. As a class, use the Pythagorean Theorem to solve for *c*, the length of the missing side, of triangle number 1. Allow students to use calculators, if you wish. If necessary, show them how to use the scientific calculator as a tool. (Note: For information about using scientific calculators, see the Department of Education manual on using scientific calculators at http://www.doe.virginia.gov/instruction/mathematics/middle/scientific_calculator_lessons.pdf .)
5. Follow the same procedure for triangle number 2.
6. Have the students solve the remaining triangles on their own.
7. When all students are finished, work together as a group to check their answers.

Reflection

Have students complete the "Reflection" worksheet.

Name: ________________________

Warm-up

Find the value of x in each of the following equations.

1. $x + 7 = 18$

2. $x - 9 = 31$

3. $x^2 = 16$

4. $x^2 = 121$

5. $x - 15 = 12$

6. $x + 4 = 21$

7. $x^2 = 64$

8. $x^2 = 81$

Name: <u>ANSWER KEY</u>

Warm-up

Find the value of x in each of the following equations.

1. $x + 7 = 18$

   $x = 11$

2. $x - 9 = 31$

   $x = 40$

3. $x^2 = 16$

   $x = 4$

4. $x^2 = 121$

   $x = 11$

5. $x - 15 = 12$

   $x = 27$

6. $x + 4 = 21$

   $x = 17$

7. $x^2 = 64$

   $x = 8$

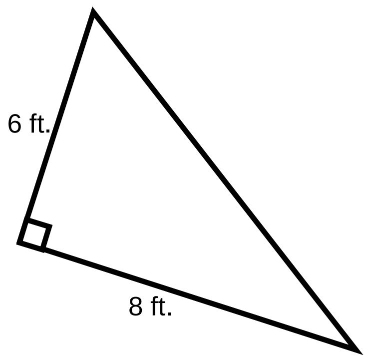
8. $x^2 = 81$

   $x = 9$

Name: ______________________

Pythagorean Theorem

Label each triangle's hypotenuse as *c* and the legs as *a* and *b*. Then, use the Pythagorean Theorem to find the value of the missing side.

1.


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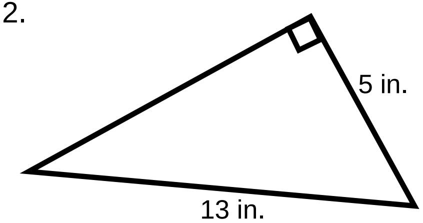
2.


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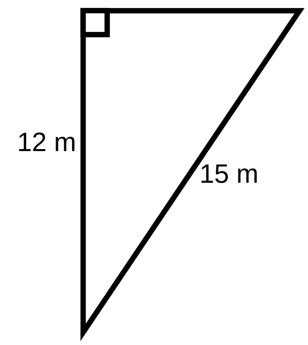
3.


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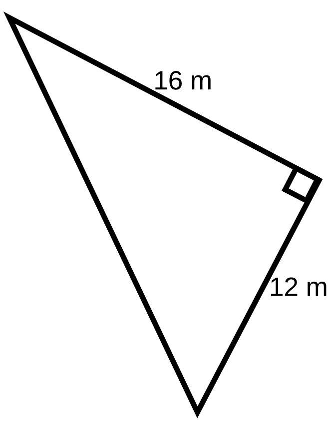
4.


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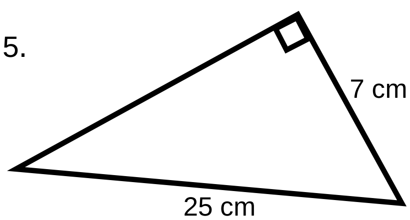
5.


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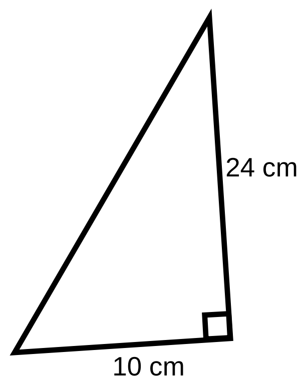
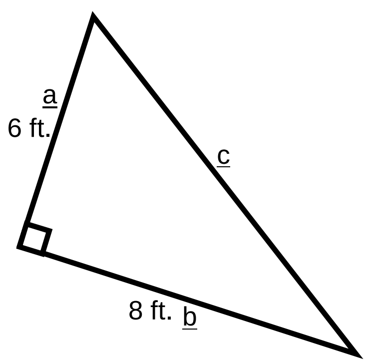
6.


____

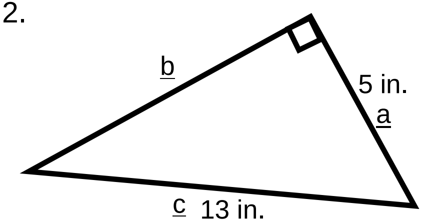
Name: <u>ANSWER KEY</u>

Pythagorean Theorem

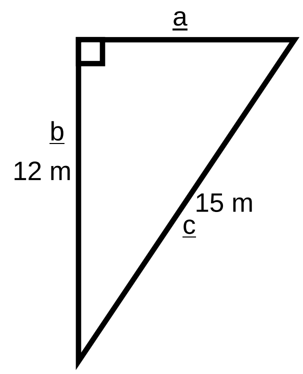
Label each triangle's hypotenuse as *c* and the legs as *a* and *b*. Then, use the Pythagorean Theorem to find the value of the missing side.

1.


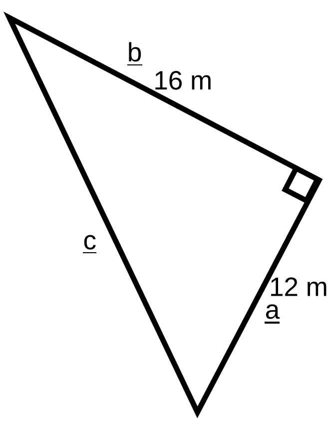
10 ft.

2.


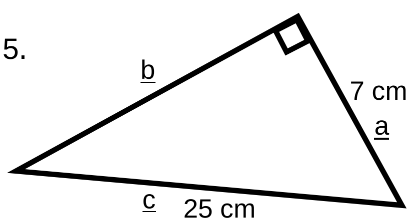
12 in.

3.


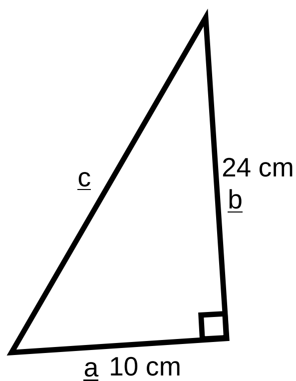
9 m

4.


20 m

5.


24 cm

6.


26 cm

Name: ______________________

Reflection

The question shown at right is a released SOL test question.

1. What type of triangle is shown?

   ____

2. Label the sides of the triangle with the terms *leg* and *hypotenuse*.

3. Write the equation you would use to solve this problem.

   ____

4. Find the value of the missing side, using the equation from step 3.

   ____

   ____

   ____

   ____

5. Which answer do you choose?

   ______

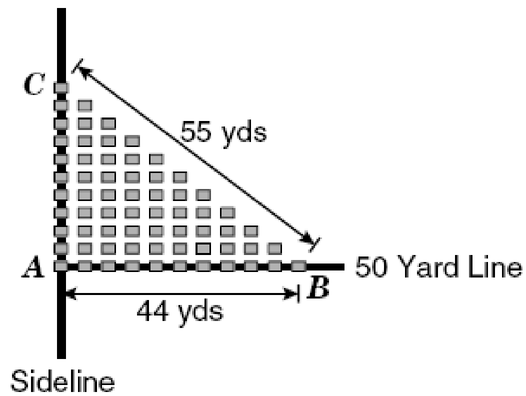
6. Explain how you decided on your answer choice.

   ____

   ____

   ____

Margo is designing a band formation for a halftime ceremony at a football game. This drawing shows where the band members will stand during the ceremony.



How many yards apart are the band members standing at points *A* and *C*?

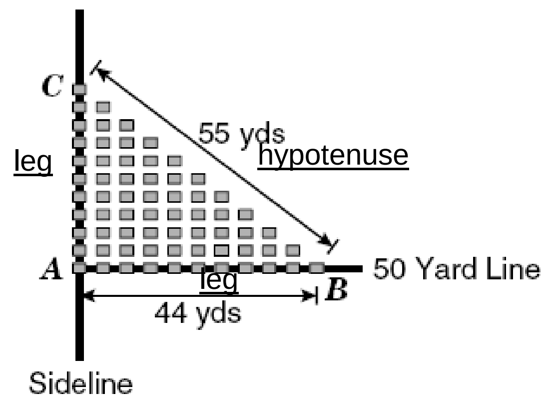
A 11
B 33
C 44
D 55

Name: <u>ANSWER KEY</u>

Reflection

The question shown at right is a released SOL test question.

Margo is designing a band formation for a halftime ceremony at a football game. This drawing shows where the band members will stand during the ceremony.



How many yards apart are the band members standing at points *A* and *C*?

A 11
B 33
C 44
D 55

1. What type of triangle is shown?

   <u>A right triangle</u>

2. Label the sides of the triangle with the terms *leg* and *hypotenuse*.

3. Write the equation you would use to solve this problem.

   $\underline{a^2 + b^2 = c^2}$

4. Find the value of the missing side, using the equation from step 3.

   $\underline{a^2 + 44^2 = 55^2}$

   $\underline{a^2 + 1{,}936 = 3{,}025}$

   $\underline{a^2 = 3{,}025 - 1{,}936}$

   $\underline{a^2 = 1{,}089}$

5. Which answer do you choose?

   <u>B</u>

6. Explain how you decided on your answer choice.