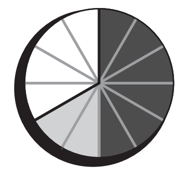
Balancing Equations



Time
30 to 40 minutes

Topic
The key to solving linear equations is realizing that both sides must "balance," so that whatever is done to one side of an equation must be done to the other as well.

Problem
Solve the linear equations presented graphically.

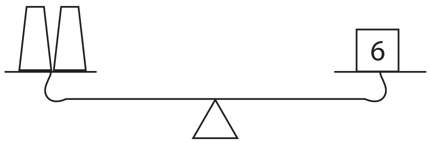
National Math Teaching Standards
- Algebra: understanding the meaning of equivalent forms of expressions
- Algebra: using symbolic algebra to represent and explain mathematical relationships
- Algebra: judging the meaning, utility, and reasonableness of the results of symbol manipulations
- Representations: selecting, applying, and translating among mathematical representations to solve problems

INTRODUCTION
This is a collection of basic linear equations in one and two variables expressed as positive quantities on either side of a simplified balance. They are designed to be solved visually. (Afterward, the standard notation of algebra could be used.)

PROCEDURE AND ANALYSIS

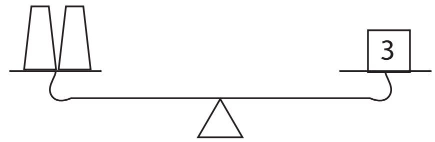
Q1. Study and solve the linear equations indicated by the simplified balances. Balance the weights on the two sides.

a.



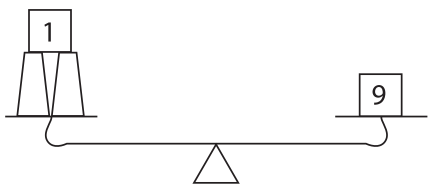
solution:  = ________

b.



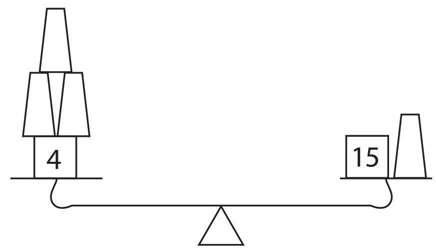
solution:  = ________

c.



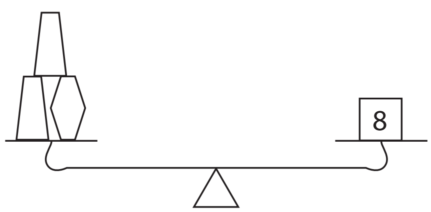
solution:  = ________

d.

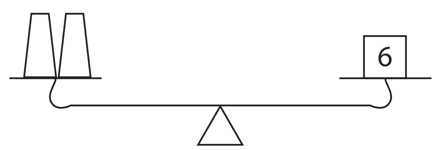


solution:  = ________

e.



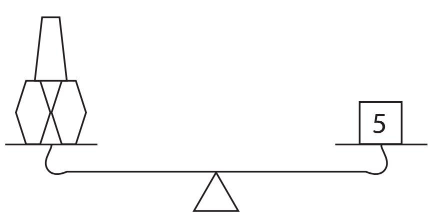
and



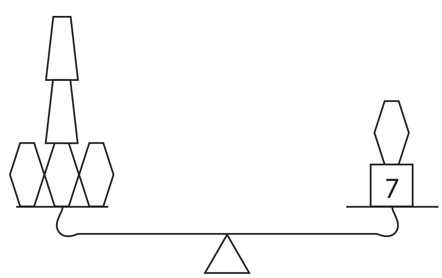
solution:  = ________

solution:  = ________

f.



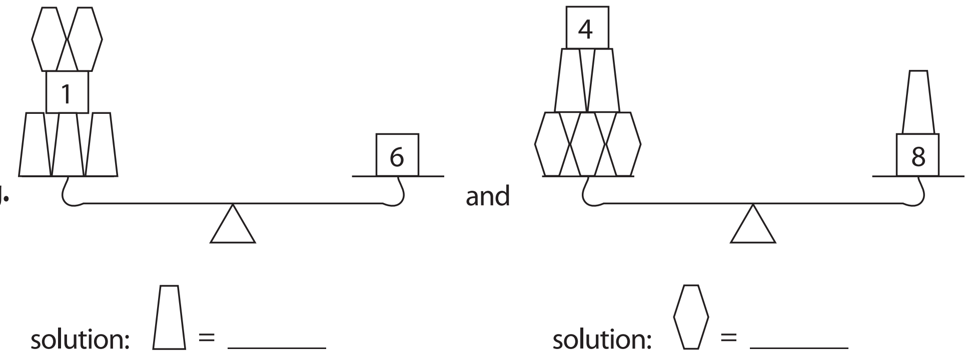
and



solution:  = ________

solution:  = ________

g.



Q2. Begin with linear equations in one variable. Sketch a corresponding balance and solve the equation.

a. $4x + 1 = 13$
b. $3x + 12 = 5x$
c. $x + 12 = 5x + 10$

Q3. Begin with a system of two equations in two variables. Sketch balances and solve the system.

$$x + 3y = 9$$

$$2x + y = y + 3$$

Q4. (Extension) Devise a way to represent negative quantities so that you can use them in the balance. Create some equations with negative quantities.

Click here to see typical answers.

2.04. BALANCING EQUATIONS

Let x represent the trapezoid.

Q1. **a.** $2x = 6, x = 3$

b. $2x = 3, x = 1.5$

c. $2x + 1 = 9, x = 4$

d. $3x + 4 = 15 + x, 2x = 11, x = 5\frac{1}{2}$

In these, x is still the trapezoid. Let y be the elongated hexagon.

e. $2x + y = 8,\ 2x = 6,\ x = 3,\ y = 2$

f. $x + 2y = 5,\ 2x + 3y = y + 7,\ x = 2,\ y = 1.5$

g. This system may be more difficult to solve visually. The first step is easy, but the rest may require the use of algebraic notation, unless the student sees that five objects balance a 5 and four objects balance a 4, so that each object balances a 1.

$$3x + 2y + 1 = 6,\ 3x + 2y = 5$$

so

$$2x + 3y + 4 = x + 8,\ x + 3y = 4$$

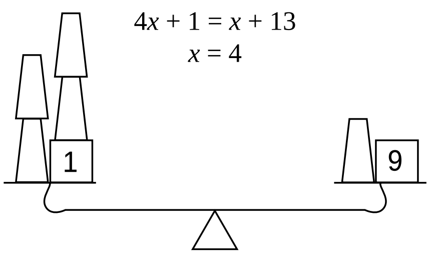
$$x = 1,\ y = 1$$

Q2.

a.

$4x + 1 = x + 13$

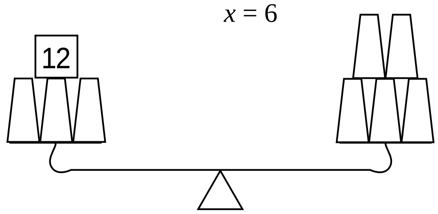
$x = 4$



b.

$3x + 12 = 5x$

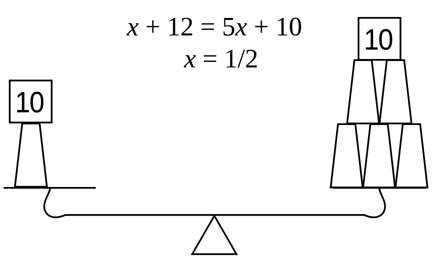
$x = 6$



c.

$x + 12 = 5x + 10$

$x = 1/2$



Q3. $x + 3y = 9$, $2x + y = y + 3$, $x = 1.5$, $y = 2.5$

Q4. One way to represent negative quantities is with a "helium-filled balloon" suspended above the balance, pulling it up. So, for example, we might have the situation pictured below.